Quintessential Adjustment
of the Cosmological Constant

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Abstract

We construct a time dependent adjustment mechanism for the cosmological "constant" which could be at work in a late Friedmann-Robertson-Walker universe dominated by quintessence and matter. It makes use of a Brans-Dicke field that couples to the evolving standard-model vacuum energy density. Our explicit model possesses a stable late-time solution with a fixed ratio of matter and field energy densities. No fine tuning of model parameters or initial conditions is required.

1   Introduction

The extraordinary smallness of the observational upper bound on the cosmological constant conflicts with naive field theoretic expectations. This is a well-known fundamental problem in our understanding of the interplay between gravity and the known quantum field theories (see, e.g., [1] for a review). Recent observations [2] suggesting a small non-zero value for the cosmological constant make this problem even more severe since an exact zero, possibly the result of a yet unknown symmetry, is replaced by a small number, $\epsilon_{vac}/M_p^4 \simeq 2 \times 10^{-123}$. In any fundamental unified theory this number would have to be calculable.

A possible alternative is a homogeneous contribution to the energy density of the universe which varies with time. It is typically connected to a time varying scalar field – the cosmon – which relaxes asymptotically for large time to zero potential energy [3,4]. The late time behavior in these models of "quintessence" [5] is insensitive to the initial conditions due to the stable attractor properties of the asymptotic solution. The homogeneous fraction of the cosmic energy density may be constant or slowly increase with time (say from 0.1 during nucleosynthesis to 0.7 today) [3,6], in sharp contrast to a cosmological constant, which needs to be adjusted such that it becomes important precisely today.

It has been argued [3] that the relaxation to a zero value of the effective potential rather than to a constant is connected to the dilatation anomaly. In absence of a fundamental theory it is, however, not obvious how to verify (or falsify) this assertion. We explore here an alternative possibility, namely that the value to which the effective cosmon potential relaxes is itself governed by a field which dynamically adjusts the "cosmological constant" to zero. We consider our proposal as an existence proof that such a mechanism can work. It is conceivable that simpler and more elegant models can be found once the basic adjustment mechanism is identified.

Rubakov has recently suggested a mechanism for the dynamical adjustment of the cosmological constant to zero [7] that avoids Weinberg's no-go-theorem [1] in a very interesting way (cf. [8] for other recent work on adjustment mechanisms). In his scenario, a scalar field governing the value of the cosmological constant rolls down a potential and approaches the zero of the potential, i.e., the point where the cosmological constant vanishes, as $t \to \infty$. Such behavior is realized by a diverging kinetic term [9], which depends on a second scalar field. This field, a Brans-Dicke-field that couples to the current value of the cosmological constant, ensures the stability of the solution.

However, in Rubakov's model the universe is inflating after the adjustment of the cosmological constant. This makes it necessary to add a period of reheating, which implies the need to fine tune the minimum of the inflaton potential to zero. In this sense, the fine tuning problem for the total effective potential is now shifted to the inflaton sector. Furthermore, it is difficult to imagine testable phenomenological consequences of such an adjustment at inflation.

The present paper suggests a dynamical adjustment mechanism for the cosmological constant that can be at work in a realistic, late Friedmann-Robertson-Walker universe. In

this model, the energy density is dominated by non-standard-model dark matter together with a cosmon field φ. This field can account for a homogeneous part of the total energy density which does not participate in structure formation and may lead to an accelerated expansion of the universe today. As in Rubakov's scenario, the cosmological constant, characterized by a field χ, rolls down a potential and approaches zero asymptotically. This is realized by a kinetic term for χ that depends on φ and diverges as t^4 when $\varphi(t) \to \infty$ at large t. To make this solution insensitive to changing initial conditions, a Brans-Dicke field σ is introduced. This field 'feels' the current value of the cosmological constant and provides the required feedback to the diverging kinetic term.

Thus, a realistic, late cosmology with an asymptotically vanishing cosmological constant arises. Baryons can be added as a small perturbation and do not affect the stability of the solution. In the concrete numerical example provided below, their coupling to the Brans-Dicke field σ is not yet realistic.

In the following, the cosmological model outlined above is explicitly constructed.

2   Adjusting a scalar potential to zero in a given Friedmann-Robertson-Walker background

The action of the present model can be decomposed according to

$$S = S_E + S_{SF} + S_{SM} \,, \tag{1}$$

where S_E is the Einstein action, S_{SF} the scalar field action, and S_{SM} the standard model action, which is written in the form

$$S_{SM} = S_{SM}[\psi, g_{\mu\nu}, \chi] = \int d^4x \sqrt{g}\, \mathcal{L}_{SM}(\psi, g_{\mu\nu}, \chi) \,. \tag{2}$$

Here $g = -\det(g_{\mu\nu})$ and ψ stands for the gauge fields, fermions and non-singlet scalar fields of the standard model (or some supersymmetric or grand unified extension). The scalar singlet χ is assumed to govern the effective UV-cutoffs of the different modes of ψ, thereby influencing the effective cosmological constant. Units are chosen such that $M^2 = (16\pi G_N)^{-1} = 1$.

Integrating out the fields ψ, one obtains (up to derivative terms)

$$S_{SM} = \int d^4x \sqrt{g}\, V(\chi) \,. \tag{3}$$

Let the potential $V(\chi)$ have a zero, $V(\chi_0) = 0$ with $\alpha = V'(\chi_0)$, and rename the field according to $\chi \to \chi_0 + \chi$. Then the action near $\chi = 0$ becomes

$$S_{SM} = \int d^4x \sqrt{g}\, \alpha\chi \,. \tag{4}$$

Due to this potential the field χ will decrease (for $\alpha > 0$) during its cosmological evolution. It can be prevented from rolling through the zero by a diverging kinetic term [7].

First, let the geometry be imposed on the system, i.e., assume a flat FRW universe with Hubble parameter $H = (2/3)\, t^{-1}$, independent of the dynamics of χ. With a kinetic Lagrangian

$$\mathcal{L}_{SF} = \frac{1}{2}\, \partial^\mu \chi \partial_\mu \chi \, F(t)\,, \tag{5}$$

one obtains the following equation of motion for χ:

$$\ddot{\chi} + (3H + \dot{F}/F)\dot{\chi} + (\partial V/\partial\chi)/F = 0\,. \tag{6}$$

Here $F(t) = t^4$ is an externally imposed condition which will be realized below by the quintessence field, which serves as a clock. Equation (6) has the particular solution $\chi = (\alpha/6)\, t^{-2}$, which provides an acceptable late cosmology since all energy densities associated with χ scale as t^{-2}.

Clearly, it requires fine tuning of the initial conditions to achieve the desired behavior $\chi \to 0$ as $t \to \infty$, which is realized in this particular solution. However, this fine tuning can be avoided by adding a Brans-Dicke field σ that 'feels' the deviation of χ from zero and provides the appropriate 'feedback' to the kinetic term so that χ reaches zero asymptotically independent of its initial value.

The field σ has a canonical kinetic term and it is coupled to $\mathcal{L}_{SM}$ by the substitution $g_{\mu\nu} \to g_{\mu\nu}\sqrt{\sigma}$ in Eq. (2),

$$S_{SM} = S_{SM}[\psi, g_{\mu\nu}\sqrt{\sigma}, \chi] = \int d^4x\, \sigma\sqrt{g}\, \mathcal{L}_{SM}(\psi, g_{\mu\nu}\sqrt{\sigma}, \chi)\,. \tag{7}$$

Integrating out the fields ψ, one obtains now

$$S_{SM} = \int d^4x\sqrt{g}\, \alpha\sigma\chi \tag{8}$$

near $\chi = 0$. The scalar field Lagrangian is now taken to be

$$\mathcal{L}_{SF} = \frac{1}{2}(\partial\chi)^2 F(\sigma,t) + \frac{1}{2}(\partial\sigma)^2 - \beta\sigma t^{-2}\,, \tag{9}$$

where $F(\sigma,t) = \sigma^2 t^4$. The additional t dependence of the term $\sim \beta$ is again introduced ad hoc and will later on be realized by the dynamics of the cosmon.

Now Eq. (6) is supplemented with the equation of motion for σ,

$$\ddot{\sigma} + 3H\dot{\sigma} + \alpha\chi - \beta t^{-2} - \sigma\dot{\chi}^2 t^4 = 0\,. \tag{10}$$

The combined equations have the asymptotic solution $\chi = \chi_0\, t^{-2}$ and $\sigma = \sigma_0 = \text{const.}$ with $\chi_0 = 3\beta/\alpha$ and $\sigma_0 = \alpha^2/(18\beta)$. The last term in Eq. (9) was introduced to allow this solution. The above solution is stable, i.e., for a range of initial conditions one still finds the desired asymptotic behavior $\chi \sim t^{-2}$ and $\sigma \sim \text{const.}$ for $t \to \infty$. This is easy to check numerically setting, e.g., $\alpha = \beta = 1$. The stability does not depend on the precise values of these parameters.

Thus, an asymptotic decay of the energy densities associated with the fields χ and σ, which is sufficiently fast to be consistent with a FRW cosmology, is realized without any fine tuning. What remains to be done is the replacement of the various t-dependent functions by the dynamics of appropriate fields.

3 Adding matter, quintessence, and gravitational dynamics

To embed the above adjustment mechanism in a realistic universe that includes matter and gravitational dynamics, assume first that the energy densities associated with χ and σ remain small compared to the total energy density throughout the evolution. The total energy density is taken to consist of $\sim 20\%$ dark matter and $\sim 80\%$ quintessence. The cosmon field can be used to realize the explicit t dependence in Eq. (9).

It is known that a system with matter and a scalar field φ that is governed by an exponential potential $V_Q(\varphi) = e^{-a\varphi}$ gives rise to a realistic late cosmology with a fixed ratio of matter and field energy densities [3–5]. The differential equations describing the system read

$$\ddot{\varphi} + 3H\dot{\varphi} + V_Q'(\varphi) = 0 \tag{11}$$

$$6H^2 = \rho + \frac{1}{2}\dot{\varphi}^2 + V_Q(\varphi) \tag{12}$$

$$\dot{\rho} + 3H\rho = 0\,, \tag{13}$$

where ρ is the density of dark matter. One finds the stable solution $H = (2/3)\,t^{-1}$, $\varphi = (2/a)\ln t$ and $\rho = \rho_0 t^{-2}$. For $a^2 = 2$ one has $\rho_0 = 2/3$, which corresponds to a realistic dark matter to quintessence ratio. The explicit time dependence in Eq. (9) can now be replaced by a coupling to φ. Technically, this is realized by the substitution $t^2 \to e^{a\varphi}$ in $\mathcal{L}_{SF}$.

Now the χ–σ system of the last section and the φ–ρ–gravity system described above have to be combined and it has to be checked whether the stability of each separate system suffices to ensure the stability of the complete system.

The complete Lagrangian, including the curvature term and the effective standard model action, Eq. (8), reads

$$\mathcal{L} = R + \frac{1}{2}(\partial\chi)^2 F(\sigma,\varphi) + \frac{1}{2}(\partial\sigma)^2 + \frac{1}{2}(\partial\varphi)^2 + V(\chi,\sigma,\varphi)\,, \tag{14}$$

where

$$F(\sigma,\varphi) = \sigma^2\, e^{2a\varphi} \qquad \text{and} \qquad V(\chi,\sigma,\varphi) = \alpha\sigma\chi + (1 - \beta\sigma)\, e^{-a\varphi}\,. \tag{15}$$

In a flat FRW universe, it gives rise to the equations of motion

$$\ddot{\chi} + (3H + \dot{F}/F)\dot{\chi} + \frac{1}{F}\frac{\partial V}{\partial \chi} = 0 \tag{16}$$

$$\ddot{\sigma} + 3H\dot{\sigma} + \frac{\partial}{\partial\sigma}\left(V - \frac{1}{2}\dot{\chi}^2 F\right) = 0 \tag{17}$$

$$\ddot{\varphi} + 3H\dot{\varphi} + \frac{\partial}{\partial\varphi}\left(V - \frac{1}{2}\dot{\chi}^2 F\right) = 0 \tag{18}$$

$$6H^2 = \frac{1}{2}\dot{\chi}^2 F + \frac{1}{2}\dot{\sigma}^2 + \frac{1}{2}\dot{\varphi}^2 + V + \rho \tag{19}$$

$$\dot{\rho} + 3H\rho = 0\,. \tag{20}$$

They have the asymptotic solution

$$\chi = \chi_0 t^{-2} \,, \quad \sigma = \sigma_0 \,, \quad \varphi = \varphi_0 + (2/a) \ln t \,, \quad \rho = \rho_0 t^{-2} \,, \quad H = (2/3) t^{-1} \,. \qquad (21)$$

For the parameters $\alpha = \beta = 1$ and $a^2 = 2$, one finds

$$\chi_0 = \frac{3}{c} \,, \quad \sigma_0 = \frac{1}{18c} \,, \quad \varphi_0 = \frac{\ln c}{\sqrt{2}} \,, \quad \rho_0 = \frac{5}{3} - \frac{1}{c} - \frac{1}{6c^2} \,, \qquad (22)$$

where $c = \left(1 + \sqrt{11/9} \right)/2$. The clustering part of the energy density is $\rho/6H^2 \simeq 0.21$.

We have checked numerically that the above solution is stable, i.e., that a small variation of the initial conditions does not affect the asymptotic behavior for $t \to \infty$. The stability does not depend on the precise values of the parameters α, β and a.

A small amount ($\sim 1\%$) of baryons can be introduced as a perturbation. In the present context, baryons are quite different from the above non-standard-model dark matter since, by virtue of Eq. (7), they couple to the Brans-Dicke field σ. Thus, the baryonic energy density is $\sim \sigma\, n_B$, where n_B is the number density of baryons. It has been checked that such an additional term does not affect the stability of the above solution. Solar system tests of the post-Newtonian approximation to general relativity place an upper bound on the coupling of baryons to almost massless scalar fields. In the present setting, the relevant coupling depends on c whose present numerical value is not compatible with phenomenology in this respect. However, this can probably be avoided in a more carefully constructed model or by the ad-hoc introduction of a kinetic term for σ that grows for large t. We also have not yet implemented the desirable slow decrease of $\rho/6H^2$, which would influence the detailed dynamics.

4   Conclusions

In the present letter, a dynamical adjustment mechanism for the cosmological constant is constructed. It can be at work in a late FRW universe and ensures that the cosmological constant vanishes asymptotically. The existence of a working late-time adjustment mechanism is interesting because of its possible observational consequences.

The field theoretic model that is used to realize this adjustment mechanism is generic but relatively complicated. In particular, it involves a Brans-Dicke field which, for the set of parameters used above, couples too strongly to standard model matter. However, there seems to be no reason why, with a different choice of parameters or scalar potentials, it should not be possible to avoid this phenomenological problem. A systematic investigation of such possibilities requires the analytic understanding of the stability of the system – a task that appears to be relatively straightforward.

The beauty of adjustment mechanisms lies in the fact that they are independent of all the intricacies of the field theoretic standard model vacuum. As a new ingredient [7] we use time or, equivalently, the value of the cosmon field as an essential parameter. Recently suggested adjustment mechanisms with an extra dimension [10] are related to

this idea since they also employ a new parameter, the position in the extra dimension, and adjust the cosmological constant to zero only at a certain value of this parameter.

One may hope that this new approach to the construction of adjustment mechanisms will eventually lead to a completely realistic and testable model.

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